AP® Calculus BC
2003 Sample Student Responses
Form B

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A₁

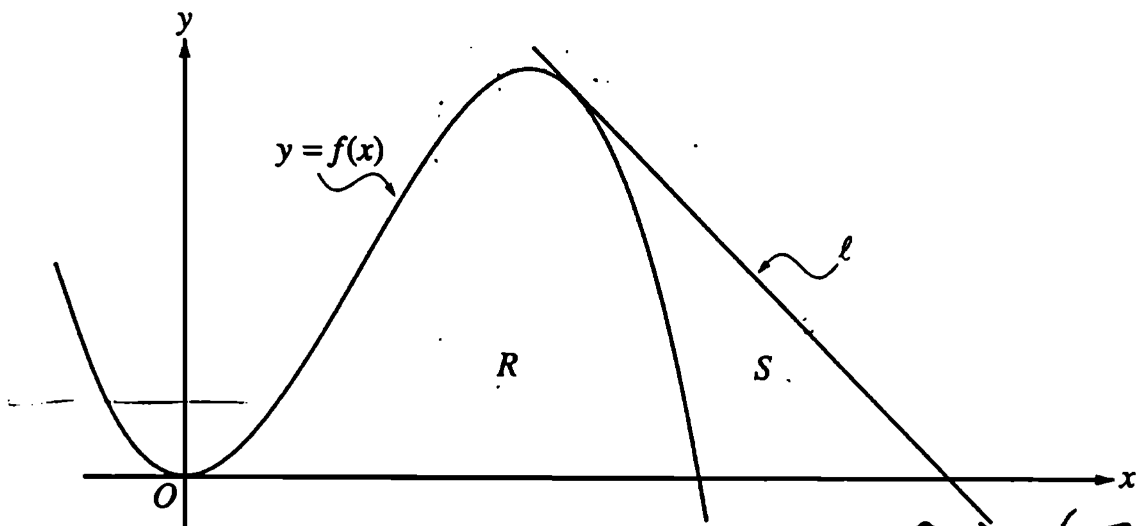
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



points: $f(x) = (3, 9)$
$\ell = (3, 9)$

Work for problem 1(a)

Value for f(x) at x = 3

$4x^2 - x^3$

$4(9) - 27$

$36 - 27$

$= 9$

Value for ℓ at x = 3

$18 - 3x$

$18 - 9$

$= 9$

the points have the same slope and the same point, so they are tangent

slope of f (3)

$f'(3) = 8x - 3x^2$

$24 - 27$

$= -3$

slope of ℓ = -3

Continue problem 1 on page 5.

A2

Work for problem 1(b)

$f(x) = 0 = 4x^2 - x^3$

$4 - x = 0$

$x = 4$

$$\left[\int_3^6 18 - 3x\right] - \left[\int_3^4 4x^2 - x^3\right]$$

$$\left\{18x - \frac{3}{2}x^2\Big]_3^6\right\} - \left\{\frac{4}{3}x^3 - \frac{1}{4}x^4\Big]_3^4\right\}$$

$(54 - 40.5) - (21\tfrac{1}{3} - 15.75)$

$(13.5) - (5.583) = $ **7.917 units²**

Work for problem 1(c)

$$\pi \int_0^4 (4x^2 - x^3)^2$$

$(4x^2 - x^3)(4x^2 - x^3)$

$16x^4 - 4x^5 - 4x^5 + x^6$

$$= \pi \int_0^4 16x^4 - 8x^5 + x^6$$

$$= \pi\left(\frac{16}{5}x^5 - \frac{8}{6}x^6 + \frac{1}{7}x^7\Big]_0^4\right)$$

$= \pi\,(156.038)$

$=$ **490.208 units³**

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B,

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

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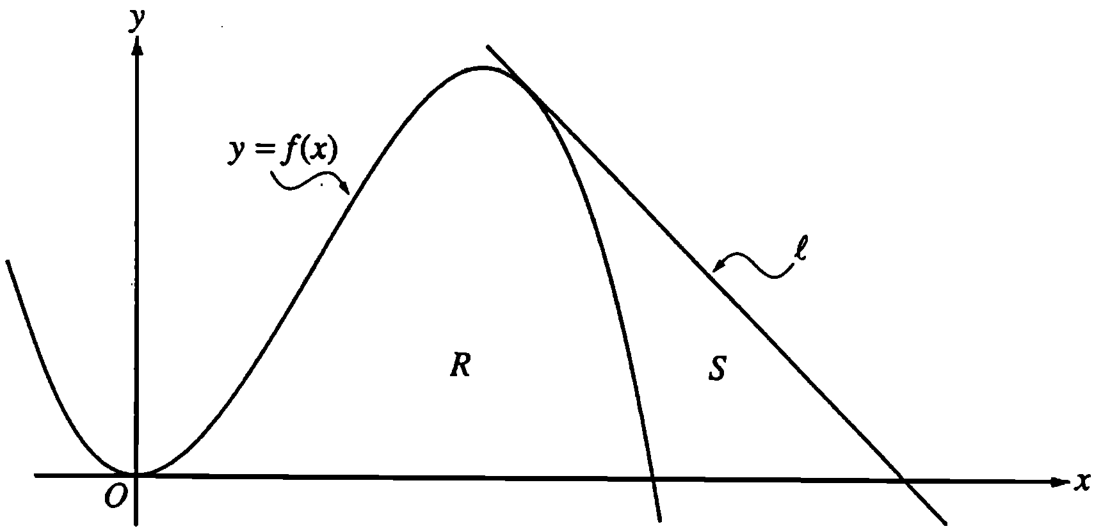

Work for problem 1(a)

Both equations have to have same value at 3.

$f(x) = 4x^2 - x^3$ $x=3$

$f(3) = 4(3)^2 - 3^3$

$= 36 - 27$

$= 9$ when $x=3$, $y=9$

$y = 18 - 3x$ $x=3$

$y = 18 - 3(3)$

$y = 18 - 9$

$= 9$ when $x=3$, $y=9$

So ℓ is tangent to the graph of $y=f(x)$ at the point $x=3$.

Continue problem 1 on page 5.

B2

Work for problem 1(b)

$f(x) = 4x^2 - x^3$

$0 = 4x^2 - x^3$

$= x^2(4 - x)$

$x = 0, 4$

$y = 18 - 3x$

$0 = 18 - 3x$

$-18 = -3x$

$x = 6$

$S = \int_3^4 [(18 - 3x) - (4x^2 - x^3)]dx + \int_4^6 (18 - 3x)dx$

$= 1.917 + 6$

$= 7.917$

Work for problem 1(c)

$V = \pi \int_0^4 (4x^2 - x^3)^2 dx$

$= \pi \cdot 156.04$

$= 490.21$

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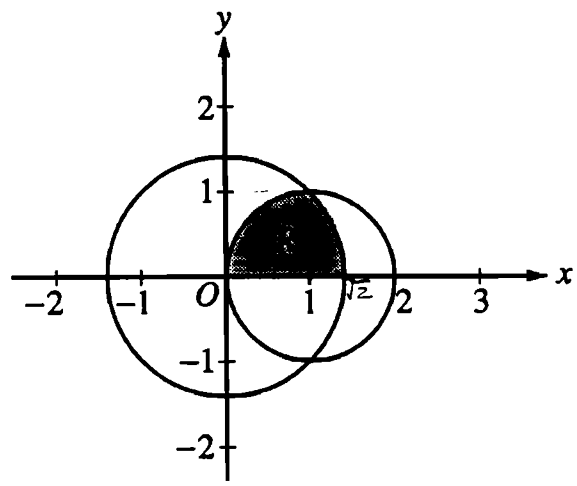


Work for problem 2(a)

We can say that, $x^2+y^2=2 \Rightarrow y=\sqrt{2-x^2}\ (y>0)$

$(x-1)^2+y^2=1 \Rightarrow y=\sqrt{1-(x-1)^2}=\sqrt{2x-x^2}\ (y>0)$

$$\therefore R=\int_0^1 \sqrt{2x-x^2}\,dx+\int_1^{\sqrt{2}} \sqrt{2-x^2}\,dx$$

(Note that $y=0$ at $x=\sqrt{2}$, in the graph of circle $x^2+y^2=2$.)

Work for problem 2(b)

We can say that $\begin{cases} x^2+y^2=2 \Rightarrow x=\sqrt{2-y^2}\ (x>0) \\ (x-1)^2+y^2=1 \Rightarrow x=1-\sqrt{1-y^2}\ (x<1) \end{cases}$

$$\therefore R=\int_0^1 \sqrt{2-y^2}\,dy-\int_0^1 (1-\sqrt{1-y^2})\,dy=\int_0^1 \{\sqrt{2-y^2}-(1-\sqrt{1-y^2})\}\,dy$$

Continue problem 2 on page 7.

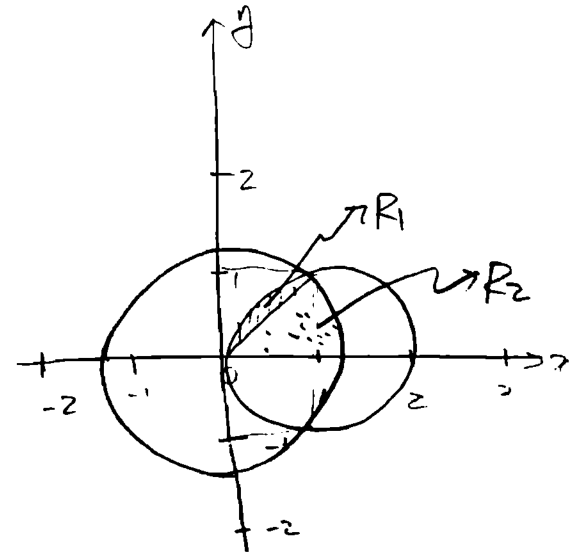
H2

Work for problem 2(c)

Let $\begin{pmatrix} r_1 = \sqrt{2} \\ r_2 = 2\cos\theta \end{pmatrix}$

The graph of r_2 goes through $(1, 1)$ when $\theta = \frac{\pi}{4}$ $\left(\because \sqrt{1^2+1^2} = 2\cos\frac{\pi}{4} = \sqrt{2}\right)$

Also, as θ increases from $\frac{\pi}{4}$ to $\frac{\pi}{2}$ on the graph of r_2, r_2 draws the arc of R_2 shown the figure below.



$$\therefore S = \int_0^{\frac{\pi}{4}} \frac{1}{2} r_1^2 \, d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} r_2^2 \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\cos^2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1 + \cos 2\theta) \, d\theta$$

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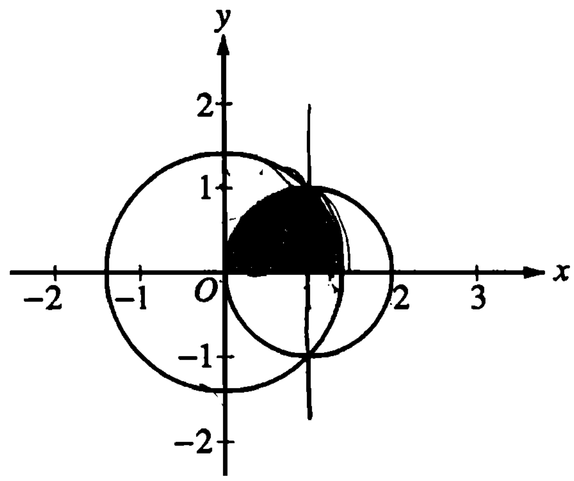

Work for problem 2(a)

$(x-1)^2 + y^2 = 1 \Rightarrow y^2 = 1-(x-1)^2 \Rightarrow y = +\sqrt{1-(x-1)^2}$

$\int_0^1 \sqrt{1-(x-1)^2}\,dx + \int_1^{\sqrt{2}} \sqrt{2-x}\,dx = 0.785 + 0.368 = 1.153$

$x^2 + y^2 = 2 \quad y^2 = 2-x$

$y = +\sqrt{2-x}$

Work for problem 2(b)

$x^2 + y^2 = 2$

$\rightarrow x^2 = 2-y^2$

$\rightarrow x = \sqrt{2-y^2}$

$\int_0^1 \sqrt{2-y^2} - (-\sqrt{1-y^2}+1)\,dy = 1.071$

$(x-1)^2 + y^2 = 1$

$\rightarrow (x-1)^2 = 1-y^2$

$\rightarrow x-1 = -\sqrt{1-y^2}$

$x = -\sqrt{1-y^2}+1$

Continue problem 2 on page 7.

Work for problem 2(c)

$S = \frac{1}{2}\int r^2\,\theta \qquad \frac{1}{2}\int (2\cos\theta)^2\,d\theta$

$R = \frac{1}{2}\int_{0.817}^{1.5} (2\cos\theta)^2\,d\theta + \frac{1}{2}\int_{0}^{0.817} (\sqrt{2})^2\,d\theta$

$= 0.255 + 0.817$

$= 1.072$

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Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

Work for problem 3(a)

Since radius $= \frac{1}{2}$ (diameter)

$\Rightarrow$ Average radius $= \frac{1}{2}\left(\frac{1}{(360-0)\text{mm}}\int_0^{360} B(x)\,dx\right) = \frac{1}{720\text{mm}}\int_0^{360} B(x)\,dx$

Work for problem 3(b)

$\int_0^{360} B(x)\,dx = \lim_{\|P\|\to 0} \sum_{k=1}^{3} f(c_k)\,\Delta x$

$c_1 = 60\text{mm} \Rightarrow f(c_1) = 30\text{mm}$

$c_2 = 180\text{mm} \Rightarrow f(c_2) = 30\text{mm}$

$c_3 = 300\text{mm} \Rightarrow f(c_3) = 24\text{mm}$

$\Delta x = \frac{360\text{mm}}{n} = \frac{360\text{mm}}{3} = 120\text{mm}$

$\Rightarrow \sum_{k=1}^{3} f(c_k)\,\Delta x = 120\text{mm}\left(f(c_1) + f(c_2) + f(c_3)\right)$

$= 120\text{mm}\,(30\text{mm} + 30\text{mm} + 24\text{mm}) = 10080\text{mm}^2$

$\Rightarrow$ Average radius $= \frac{1}{720}\int_0^{360} B(x)\,dx \approx \frac{1}{720\text{mm}}\left(10080\text{mm}^2\right) = 14\text{mm}$

Continue problem 3 on page 9.

Work for problem 3(c)

It is the volume of blood in the blood vessel starting from a distance of 125 mm from 1 end to a distance of 275 mm from the same end. The units will be $(mm)^3$

Work for problem 3(d)

$b, a, d, c, e, f \in (0, 360)$

$c > f$, $d > e$, $b > a$

$B''(x) = 0 \Rightarrow \frac{B'(b) - B'(a)}{b - a} = 0$

$\Rightarrow B'(b) = B'(a)$

$\Rightarrow \frac{B(d) - B(e)}{d - e} = \frac{B(c) - B(f)}{c - f}$

Since for all x, Δx is the same

$\Rightarrow B(d) - B(e) = B(c) - B(f)$

From the table there are values of d, e, c, f such that $B(d) - B(e) = B(c) - B(f)$. For example at $x = 300$

$B(360) - B(300) = B(300) - B(240) \Rightarrow 26 - 24 = 26 - 24 \Rightarrow 0 = 0$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

$F_{avg} = \frac{1}{b-a}\int_a^b F(x)\,dx$

Work for problem 3(a)

$B(x)_{avg} = \frac{1}{360-0}\int_0^{360} \frac{B(x)}{2}\,dx$

$= \frac{1}{360}\int_0^{360} \frac{Bx}{2}\,dx$

Work for problem 3(b)

$\frac{360}{3} = 120$

$B(x)\ avg = \frac{1}{360}\left[\frac{120f(60)}{2} + \frac{120\cdot f(180)}{2} + \frac{120\cdot f(300)}{2}\right]$

$= \frac{120}{360}\left[15 + 15 + 12\right]$

$= \frac{12}{36} \times 42$

$= 14$ mm

Continue problem 3 on page 9.

Work for problem 3(c)

$\frac{B(x)}{2}$ = radius of blood vessel

$\pi \int_{125}^{275} \left(\frac{Bx}{2}\right)^2 dx$ Volume of the blood vessel from $x = 125$ mm to $x = 275$ mm in $(mm)^3$

Work for problem 3(d)

At x where $B''(x) = 0$

There is an inflection on the graph

The sign of $B'(x)$ changes

$B'(x)$ = the change of diameter

From the table we know that when the diameter increases $B'(x) > 0$ when diameter decrease $B'(x) <$

$B'(x)$ changes signs

$\therefore B''(x) = 0$

END OF PART A OF SECTION II

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NO CALCULATOR ALLOWED

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

Velocity vector $= \left(\frac{dx}{dt}, \frac{dy}{dt}\right) = \left((2e^{3t}+e^{-7t})', (3e^{3t}-e^{-2t})'\right)$

$= \left(6e^{3t}-7e^{-7t}, 9e^{3t}+2e^{-2t}\right)$

speed $= \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} = \sqrt{(6e^{3t}-7e^{-7t})^2+(9e^{3t}+2e^{-2t})^2}$

$t=0$

$= \sqrt{(6-7)^2+(9+2)^2} = \sqrt{1+121} = \boxed{\sqrt{122}}$

Work for problem 4(b)

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \boxed{\frac{9e^{3t}+2e^{-2t}}{6e^{3t}-7e^{-7t}}}$

$\lim_{t\to\infty} \frac{dy}{dx} = \frac{9e^{3t}}{6e^{3t}}$ $\left(\because \lim_{t\to\infty} 7e^{-7t} \text{ and } 2e^{-2t} = 0\right)$

$= \boxed{\frac{3}{2}} = \boxed{1.5}$

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

horizontal tangent's $f'(x)=0=\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{9e^{3t}+2e^{-2t}}{6e^{3t}-7e^{-7t}} \neq 0$

but $9e^{3t}>0 \quad (t\in\mathbb{R})$

$2e^{-2t}>0 \quad (t\in\mathbb{R})$

so $9e^{3t}+2e^{-2t}>0 \quad (t\in\mathbb{R})$

therefore there is no horizontal tangent.

Work for problem 4(d)

$\frac{dy}{dt}=\frac{9e^{3t}+2e^{-2t}}{6e^{3t}-7e^{-7t}}$

when this part value goes 0, the value of $\frac{dy}{dx}$ increases to ∞ or $-\infty$ which is vertical tangent.

$\therefore 6e^{3t}-7e^{-7t}=0$

$6e^{3t}=7e^{-7t}$

$6e^{10t}=7$, $e^{10t}=\frac{7}{6}$

$10t=\ln\frac{7}{6}$ $\quad t=\frac{1}{10}\ln\frac{7}{6}$

$t=0.1(\ln 7-\ln 6)$

GO ON TO THE NEXT PAGE.

at this point there must be

C1

NO CALCULATOR ALLOWED

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$y'(t) = 3 \times e^{3t} \times 3 - e^{-2t} \times (-2) = 9e^{3t} + 2e^{-2t}$

$x'(t) = 6e^{3t} - 7e^{-7t}$

velocity vector for the particle $(6e^{3t} - 7e^{-7t},\ 9e^{3t} + 2e^{-2t})$

speed = |velocity| (at $t=0$) $= \left| \dfrac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}} \right| = \left| \dfrac{9+2}{6-7} \right| = 11$

Work for problem 4(b)

$$\frac{dy}{dx} = \frac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}}$$

$$\lim_{t\to\infty} \frac{dy}{dx} = \lim_{t\to\infty} \frac{9e^{3t}}{6e^{3t}} = \lim_{t\to\infty} \frac{3}{2} = \frac{3}{2}$$

Continue problem 4 on page 11.

C2

NO CALCULATOR ALLOWED

Work for problem 4(c)

The line tangent to the path of the particle that is horizontal must have a slope of 0.

Find values of t where $\frac{dy}{dx} = 0$

$9e^{3t} + 2e^{-2t} > 0$. for all values of t

because e^{3t} is always a positive value

and e^{-2t} is also always a positive value

Therefore, none exists.

Work for problem 4(d)

For the line tangent to the path of the particle to be vertical, $\frac{dy}{dx}$ must be inpinite.

$\frac{dy}{dx}$ is infinite when $6e^{3t} - 7e^{-7t} = 0$

$6e^{3t} = 7e^{-7t}$

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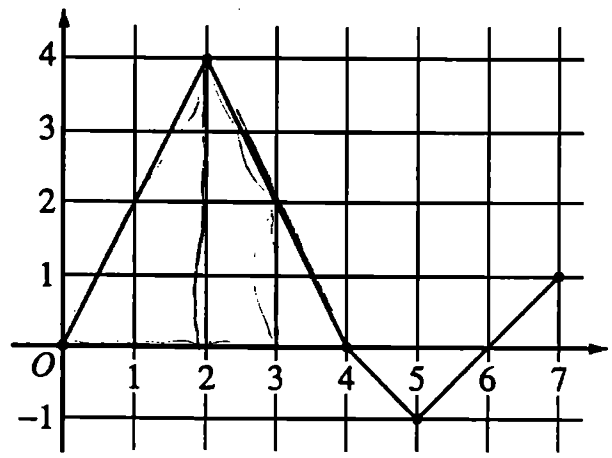


Graph of f

Work for problem 5(a)

4+3

$g(x) = \int_2^x f(t)\,dt$

$g'(x) = f(x)$

$g''(x) = f'(x)$

$g(3) = \int_2^3 f(t)\,dt = F(3) - F(2) = 7 - 4 = 3$

$g'(3) = f(3) = 2$

$g''(3) = f'(3) = -2$

$g(3) = 3$
$g'(3) = 2$
$g''(3) = -2$

Work for problem 5(b)

rate of change of $g = g'(x)$

$\frac{1}{3-0}\int_0^3 g'(x)\,dx = \frac{1}{3}\{g(3) - g(0)\}$

$= \frac{1}{3}\{3 - g(0)\}$

$= \frac{1}{3}\left(3 - \int_2^0 f(t)\,dt\right)$

$= \frac{1}{3}\left(3 + \int_0^2 f(t)\,dt\right)$

$= \frac{1}{3}\left(3 + F(2) - F(0)\right)$

$= \frac{1}{3}(3 + 4 - 0)$

$= \frac{7}{3}$

$\frac{7}{3} \simeq 2.333$

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

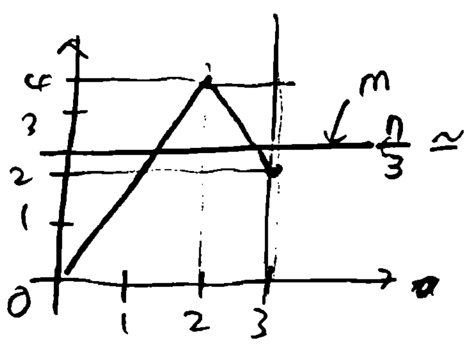
A2

Work for problem 5(c)

$g'(c) = \frac{7}{3}$

since $g'(x) = f(x)$, $g'(c) = f(c)$.

$g'(c) = f(c) = \frac{7}{3}$



The line m crosses the graph of f twice

→ $g'(c)$ is equal to 2.333 at two values of c.

Work for problem 5(d)

At points of inflection, $g''(x)$ should change from (+) to (−), or vice versa.

At $f(2)$, $f'(x)$ changes from (+) to (−), and at $f(5)$, $f'(x)$ changes from (−) to (+).

Points of inflection exist at $x=2$ and $x=5$.

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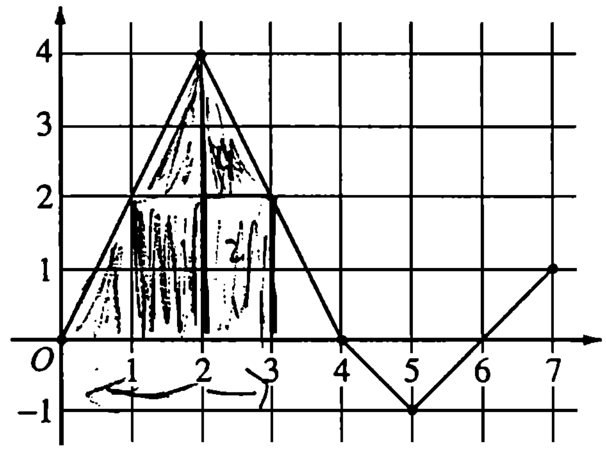


Graph of f

Work for problem 5(a)

$g(3) = \int_2^3 f(t)\,dt = \boxed{3}$

$g'(x) = f(x) \Rightarrow g'(3) = f(3) = \boxed{2}$

$g''(3) = f'(3) = \text{slope at } 3 = \frac{2-4}{3-2} = \frac{-2}{-1} = \boxed{2}$

Work for problem 5(b)

avg rate of change $= \dfrac{g(a)-g(b)}{a-b}$

$g(0) = \int_2^0 f(t) = -4$

$g(3) = \int_2^3 f(t) = 3$

$\dfrac{g(0)-g(3)}{0-3} = \dfrac{-4-3}{-3} = \boxed{\dfrac{7}{3}}$

Continue problem 5 on page 13.

Work for problem 5(c)

$g'(c) = 7/3 \Rightarrow$

$f(c) = 7/3$ at 1 (one) point

because

on (0, 2), $f(x) = y = 2x$

$2x = 7/3$

$x = 7/6$ ← only at $x = 7/6$

on (2, 3), $f(x) = y = 2x + 8$

$7/3 = 2x + 8$

$7/3 - \frac{24}{3} = 2x \Rightarrow -\frac{17}{3} = 2x$

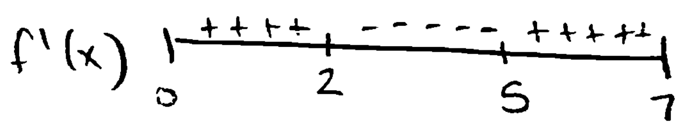
~~$x = -17/6$~~ (not on (0, 3))

Work for problem 5(d)

point of inflection $= g''(x) = 0$

$g''(x) = f'(x)$

$f'(x) = 0$ at $x = 2$, $x = 5$



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Work for problem 6(a)

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!} + (x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$$

$$\Rightarrow f(x) = 1 + \frac{2}{3}(x-2) + \frac{3}{9}(x-2)^2 + \frac{4}{27}(x-2)^3 + \ldots + \frac{n+1}{3^n}(x-2)^n + \ldots$$

Work for problem 6(b)

Using the ratio test:

$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{n+2}{3^{n+1}}(x-2)^{n+1} \cdot \frac{3^n}{n+1}\frac{1}{(x-2)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{n+2}{n+1}\cdot\frac{x-2}{3}\right| = \frac{x-2}{3}$$

$\Rightarrow$ Since $|L| < 1$ for series to converge

$$-1 < \frac{x-2}{3} < 1 \Rightarrow -1 < x < 5$$

Continue problem 6 on page 15.

$\Rightarrow$ The radius of convergence is 3.

NO CALCULATOR ALLOWED

A2

Work for problem 6(c)

$g(2) = 3$, $g^{(n)}(2) = \frac{n!}{3^{n-1}}$.

$\Rightarrow g(x) = 3 + (x-2) + \frac{1}{3}(x-2)^2 + \frac{1}{9}(x-2)^3 + \ldots + \frac{1}{3^{n-1}}(x-2)^n + \ldots$

Work for problem 6(d)

Since $T_n = \frac{(-4)^n}{3^{n-1}}$ if $x = -2$. Then,

$T_n = (-1)^n \cdot 3\left(\frac{4}{3}\right)^n$

$\Rightarrow$ alternating geometric sequence with $r = \frac{4}{3} > 1$; it is neither absolutely convergent or conditionally convergent, hence g does not converge at $x = -2$.

END OF EXAMINATION

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Work for problem 6(a)

$$f^{(n)}(2) = \frac{(n+1)!}{3^n}$$

$$f(x) = 1 + \frac{2!}{3}(x-2) + \frac{3!}{2!3^2}(x-2)^2 + \frac{4!}{3!3^3}(x-2)^3 + \cdots + \frac{n+1}{3^n}(x-2)^n$$

Work for problem 6(b)

$$f(x) = \sum_{0}^{\infty} \frac{n}{3^n}(x-2)^n$$

by ration test, $\lim_{n\to\infty} \left| \frac{(n+2)\left(\frac{x-2}{3}\right)^{n+1}}{(n+1)\left(\frac{x-2}{3}\right)^n} \right| = \lim_{n\to\infty} \left| \left(\frac{n+2}{n+1}\right)\left(\frac{x-2}{3}\right) \right| \to \left|\frac{x-2}{3}\right|$

$f(x)$ converges if $\left|\frac{x-2}{3}\right| < 1$

$|x-2| < 3$

∴ the radius of convergence of f about $x=2$ is 2

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$g(2)=3$; $g'=f(x)$

$g = \int f(x)dx = x + C + \frac{1}{3}(x-2)^2 + \frac{1}{3^2}(x-2)^3 + \frac{1}{3^3}(x-2)^4 + \dots + \frac{1}{3^n}(x-2)^{n+1}$

Work for problem 6(d)

by ratio test, $\lim_{x\to\infty} \left| \frac{\left(\frac{x-2}{3}\right)^{n+2}}{\left(\frac{x-2}{3}\right)^{n+1}} \right| \to \left|\frac{x-2}{3}\right|$

$\therefore$ g converges if $\left|\frac{x-2}{3}\right| < 1$

$-1 < x < 5$

$x=-2$ is outside of the range of convergence

$\therefore$ g is not converge at $x=-2$

END OF EXAMINATION

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